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Age Effects on Equilibrium Unemployment*

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Abstract

This paper analyzes how the demographic changes in the age composition of the labor force affect unemployment through shifts in labor turnover and job creation. A theoretical model of equilibrium unemployment is used to study the effects of age-related changes in the mean value of a job-worker match and in average separation risks. The analysis produces four regimes with different effects on unemployment and vacancies. We then examine empirically for a set of 12 OECD economies which country relates to which regime. According to the estimates we can identify all four possible outcomes. We find, for example, that the ongoing aging of the working force will cause an increase in unemployment in France and Japan but a decrease in Germany and North America.

Keywords: Vacancies and Separations, Unemployment, Job Creation, Aging of the Labor Force, Demographic Change

JEL classification: J63, J64, J23, J21, J10

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1 Introduction

The end of the baby boom and the persistent decline in fertility rates fundamentally changed the relative population shares of younger and older age groups in most developed countries. Less people younger than 40 years old faced more and more people older than 40 years old during the last 30 years. Considering those who are generally assumed to be able to work yields that the ratio of people aged 15 to 39 years old to those aged 40 to 64 years old declined between 1978 and 2004 from 1.7 to 1.0 in Canada, from 1.2 to 0.8 in Germany, from 1.3 to 0.9 in Japan and from 1.5 to 1.0 in the USA. While the mean age of the people of working age declined in most countries before the 70s, the future will bring an even accelerated aging of the labor force.

Labor market consequences of the altering age structure are discussed in different ways. Examples are the analysis of the effects of a population decline including feedbacks of the capital-labor ratio, the effects of the age structure on labor productivity, shifts in cohort-specific wage levels, and changes in goods demand which affect labor mobility (see Börsch-Supan, 2003; Johnson and Zimmermann, 1993). The effects of the age structure on unemployment are evaluated with a shift-share approach or variable age-specific unemployment rates. Shimer (1998) attributes changes in total unemployment to increases or declines in the population shares of age groups that have comparable lower or higher unemployment rates. In contrast to this Shimer (2001) and Nordström Skans (2005) consider also indirect effects which come from variable age-specific unemployment rates. In their analysis of the U.S. states and Swedish local labor markets respectively, they find a positive cohort effect which means that unemployment tends to be lower if many younger workers supply labor. However, a fall in employment in older cohorts may contrast this effect. Shimer’s (2001) basic argument for the positive employment effect of large younger cohorts is that firms react by creating more jobs if younger workers undertake more search in the labor market because they look for either their first or a better job.

The complementing side of analyzing age-related search effects in the labor market is that the age structure can affect the efficiency of matching jobs and workers. The effects can be found in some empirical studies on the matching function which include demographic variables. Coles and Smith (1996) argue in their study for England and Wales that matching decreases with an older working population. Pissarides and Wadsworth (1994) and Burgess (1993) find evidence for Great Britain that separation rates are higher for younger workers as they are more likely to undertake on-the-job search.

1Other impacts of the demographic change relevant for the labor market can be: Labor supply and the role of the social security systems (Breyer and Stolte, 2001; Ehrlich and Kim 2005), changes in consumption (Batey and Madden, 1999), and growth effects (Miles, 1999; Bloom and Canning 2004).
This paper complements the literature with its modeling of equilibrium unemployment and its subsequent empirical analysis. The novelty of our analysis is multiple. The primary innovation is that we consider differences in individual attributes of younger and older workers so that firms change their willingness to create jobs not only due to variations in the cohort sizes but also due to expected changes in employment duration and average labor productivity. Second, we argue that it is not only the population share of the youngest and the oldest age group which makes a difference. Instead, workers in their early or late prime age may differ in their employment-relevant attributes if, for example, seniority effects on productivity are significant in the first years of employment and the risk of mismatch and separation rates are higher in the worker’s first job. Third, to our best knowledge this is the first approach to estimate both the Beveridge curve and the job creation curve. The benefit of this approach is that it allows to identify all possible shifts of equilibrium unemployment due to a change in the age composition of the unemployed. Finally, we use macro data to provide a comparison between 12 OECD countries to explore whether age effects on unemployment are general or subject to country-specific economic and institutional factors.

The first task of the paper is to develop a model of equilibrium unemployment which follows the standard search models of Diamond-Mortensen-Pissarides. Age-related effects are introduced with the consideration of a productivity differential between old and young and differences in age-specific separation risks. We identify four regimes with different changes in the Beveridge curve and job creation which can occur if the age structure varies. Only in two cases we obtain clear, but opposing, results with either an increase or a decrease in unemployment as soon as the average job seeker grows old. To solve the ambiguity due to the four possible equilibrium shifts, we estimate the Beveridge curve and the job creation curve for 12 OECD countries. In addition to a considerably set of control variables we account for the effects of a changing age composition, and find all four cases of age effects. For example, Germany, Canada and the USA may experience positive employment effects from an increase in the share of workers aged 40 years old and older. In contrast to this, fewer younger workers will cause a rise in unemployment in France and Japan.

The remainder of the paper is organized as follows. In section 2 we model equilibrium unemployment under the assumption of age-related heterogeneity in the labor force. Section 3 presents the econometric model and reports the estimation results. Finally, we summarize our results in section 4.

2 The Theoretical Model

Our modeling extends the original framework of search and equilibrium unemployment (see Pissarides, 2000) with the distinction between younger and
older workers and age-related effects of job creation and job destruction.\footnote{We analyze the effects of aging of the labor force but ignore effects from a population decline. The reason for this is that most empirical studies find constant returns to scale of matching functions. Petrongolo and Pissarides (2001) provide an overview of the related literature. Therefore, the pure population size has no effect on matching and search equilibrium in the labor market.}

The standard model implies that countries with an older labor force will have lower unemployment rates. This is due to the simple assumption that young workers are born into unemployment. In contrast to this, we ignore "births" and "deaths" in the labor market but analyze the effects on equilibrium unemployment if younger and older workers differ in some individual characteristics. From this it follows that changes in the age structure can have ambiguous effects on unemployment.

The way we introduce heterogeneity into the labor force follows Acemoglu (1997), who distinguished between high-skilled and low-skilled workers. In contrast to this, we differentiate between younger and older workers who may generate different levels of surplus for firms if they fill a vacancy. We consider age-sensitive differentials in labor productivity, wages, and separation risks. The purpose of the setting is to be general enough to catch the different ways of how shifts in the age composition can affect equilibrium unemployment via job creation and job destruction. Due to the generality we can apply our theoretical results to a macroeconometric model in section 3 to analyze how the demographic change influences unemployment.

\section{Matching and Equilibrium Worker Flows}

There are two types of agents, workers and firms. All agents are risk neutral and discount the future at rate $r$. From the individual attributes $i$ of workers we consider age as the only relevant factor here. Hence the labor force is divided into two age groups which have a share of $p$ and $1 - p$ respectively. Later in the empirical part we cut the labor force within the group of prime age workers. Therefore, we henceforth denote the two groups as the younger (than the mean age) workers, $y$, and the older (than the mean age) workers, $o$. Workers are either employed or unemployed which means that they seek for a new job. The average rate of unemployment in a continuum of workers, normalized to 1, is then composed of the age-specific rates weighted at the relevant population share, $u = (1 - p) u_o + p u_y$. The share of younger workers in the unemployment pool is $p u_y$ and the one of older workers is $(1 - p) u_o$, with $\tilde{u}_i = u_i / u$ denoting the relative unemployment risk at age $i = [y, o]$. Hence, the age composition of the unemployed has a demographic and an economic element as the relative unemployment risk can be different from unity. A firm can be in one of the three states: It is inactive at zero return, it seeks for a worker at search costs, or it hires one worker, starts production and earns profits. Vacancies are equally open to younger and
older workers. We show later that we receive a pooling equilibrium indeed if search costs are equal to the benefits from waiting for a better performing younger or older worker.

Search frictions limit the matching of unemployed and vacancies. New employment relations are created through a standard matching technology which forms the number of matches from the number of unemployed workers and the number of vacancies. Hence, \( m = m(u, v) \) is the flow rate of matches formed, with \( v \) denoting the vacancy rate. As standard, \( m(u, v) \) exhibits constant returns to scale in its two arguments, is continuous and differentiable, and \( m(u, v) < \infty \). Equilibrium in search models usually critically depends on a measure of the tightness of the labor market defined as \( \theta = v/u \). This is because \( \theta \) determines how successful search is. A firm with a vacancy meets a job seeker at a rate \( q(\theta) = m(u, v)/v \), decreasing in the vacancy-unemployment ratio. We assume that the matching technology is random in the sense that if the proportion of younger workers in the unemployment pool is \( p_{e_u} \), then the conditional probability that a vacancy is filled with a younger worker is \( p_{e_u} \), too. Equivalently, a job seeker finds a new employment at rate \( \theta q(\theta) \) which is identical for both age groups as vacancies do not differentiate between younger and older candidates.

Job-worker matches have a finite time horizon. Once formed, matches have a constant risk to come to an end and the state of a firm changes to new search or inactivity. Separation takes place because of idiosyncratic shocks which hit all matches at the same probability \( s \). In addition to this, age-related shocks are possible. Let \( \tau_o \) and \( \tau_y \) denote the rates which indicate the added risk that the match ends as the worker is older or younger. The rates may also include different quitting rates, for example because of family moves etc. The age-related separation rates are added to \( s \) to define the risk of losing the joint surplus of a job-worker match.

Unemployment rates of younger and older workers evolve according to job creation and job destruction. Age-related separations risks and employment rates give the flow into unemployment, the matching rate yields the transition probability for the unemployed:

\[
\dot{u}_o = (1 - p) (s + \tau_o) (1 - u_o) - \theta q(\theta) (1 - p) u_o, \]

\[
\dot{u}_y = p (s + \tau_y) (1 - u_y) - p\theta q(\theta) u_y, \]

From \( \dot{u}_i = 0 \) it follows that the age-specific rate of equilibrium unemployment is \( u_i = (s + \tau_i) / (s + \tau_i + \theta q(\theta)) \), with \( i = [y, o] \). The summation of the two unemployment rates weighted at the respective population proportions then yields the Beveridge curve (BC):

\[
u = \frac{s}{s + \theta q(\theta)} + p (u_y - u_o) .
\]

This is the standard BC plus an age-related effect which disappears if the separation rate is identical for younger and older workers. Otherwise, an
increasing proportion of that age group with the higher separation rate increases job destruction and unemployment. Furthermore, the unknown $\theta$ in BC determines equilibrium unemployment and is explained by the willingness by the firms to create vacancies. Firms employ younger and older workers but the surplus from a match can be different. Hence, we analyze next how job creation, and therefore $\theta$, depend on the age structure.

2.2 Firms

Whether firms create new jobs or remain inactive is subject to the benefits they receive and the costs they have to pay during their market activities. The benefits and costs include the (present discounted) value of the states: Match with an older worker $J_o$, match with a younger worker $J_y$, and unfilled vacancy $V$. The values satisfy the Bellman equations

$$rJ_o = \mu - w_o - (s + \tau_o) (J_o - V),$$
$$rJ_y = \mu + \delta - w_y - (s + \tau_y) (J_y - V),$$
$$rV = -\gamma + q(\theta) (V - J).$$

Firms receive revenues $\mu$ from selling the output if an older worker is employed, while they pay the wage $w_o$ as compensation. Equivalently, a younger worker produces the value $\mu + \delta$ and earns $w_y$. Experience and lower training costs favor older workers but a lower depreciation of human capital is an argument for a higher productivity of younger workers. As it is not clear which effect dominates, we do not fix the sign of the output differential so that $\delta \geq 0$. The job-worker match ends at the probability $s + \tau_i$, in which case the value of the match is replaced by the value of an unfilled vacancy. The vacant job costs $\gamma$ per unit time and changes state according to the Poisson Process at rate $q(\theta)$. The change of state yields net return $V - J$ with $J$ denoting the expected value of a filled vacancy. As the firm can meet two types of workers, we consider that the worker is younger at the probability $p\tilde{u}_y$ and he is older at the probability $(1 - p)\tilde{u}_o$. The probabilities are equal to the relative shares of younger and older workers in the unemployment pool. Therefore, the expected value of filling the vacancy is:

$$J = p\tilde{u}_yJ_y + (1 - p)\tilde{u}_oJ_o.$$  

As revenues exceed costs in any case, a job-worker match is always more profitable than a vacant job.

---

See Börsch-Supan et al. (2005) on the difficulty of the measurement of individual age-related productivity.
2.3 Workers

Workers have an impact on the equilibrium outcome through their role in wage determination. The employment of an older or a younger worker provides different returns to the firms. For this reason, a job should bring a different income also for the two types of workers. Consequently, older workers earn \( w_o \) and younger ones \( w_y \) when employed and they receive some real return \( b \) during job search. Typically the major component of \( b \) are unemployment insurance benefits. As these payments have only little age-sensitive elements, we assume the same rate \( b \) for older and younger job seekers. Let \( U \) and \( W \) denote the present-discounted value of the expected income stream of an unemployed and an employed worker, respectively. The unemployed get benefits \( b \) and in unit time they can expect to move into employment at the probability \( \theta q(\theta) \). In this case they gain \( W \) but lose \( U \).

The chance to find an employment is equal for older and younger workers as firms do not advertise age-segregated vacant jobs. The permanent income of employed workers is different from the constant wage as the match ends for an individual at probability \( s + \tau_i \) and the status changes from \( W \) to \( U \). Hence, individuals at age \( i = [o, y] \) can expect benefits from labor supply which satisfy

\[
rU_i = b + \theta q(\theta) (W_i - U_i) \tag{8}
\]

during job search and

\[
rW_i = w_i + (s + \tau_i) (U_i - W_i) \tag{9}
\]

if they are employed. Though workers of any age can fill the same vacancy, a wage differential between \( w_o \) and \( w_y \) reflects that older and younger workers can be of different value for a firm. The separation risk also varies with age \((\tau_o \lessapprox \tau_y)\). This affects the probability of a change of state towards unemployment and consequently affects the expected values of \( W_i \) and \( U_i \).

As usual we assume that \( w_i > b \) and workers do not give up their jobs due to a higher alternative income.

2.4 Equilibrium

The final determination of the market tightness and, hence, equilibrium job creation demands two things. First, wage determination has to specify the labor costs so that firms can evaluate the actual value of filling a vacancy. And second, the age-related job-worker matches have to satisfy the pooling condition\(^4\), which leads to vacant jobs that do not distinguish between older workers.

\(^4\)A separating equilibrium either means the exclusion of one age group or the exclusion of age effects. Both results contradict the purpose of our analysis. Moreover, only few advertisements of vacancies have information about a minimum or a maximum age of successful candidates.
and younger workers. From this it follows how job creation complements the BC to establish equilibrium with \( w^*, u^*, \theta^* \).

Wages are derived from the Nash bargaining solution. The wage for older and younger workers is the rate \( w_o \) and \( w_y \) respectively that maximizes the weighted product of the worker’s and the firm’s net return from an employment. While the worker gains \( W_i \) but loses \( U_i \) if she starts a new job, the firm gives up \( V \) for \( J \). The share of the total benefits that each party receives depends on a measure \( \beta \) which is usually interpreted as the bargaining power of the workers. As shown in Appendix, the bargaining outcome yields the age-sensitive wages:

\[
\begin{align*}
  w_o &= \frac{(1 - \beta) b + \beta \left[ \mu + \theta q(\theta) \frac{\mu}{r + s + \tau_o} \right]}{1 + \frac{\beta \theta q(\theta)}{r + s + \tau_o}}, \\
  w_y &= \frac{(1 - \beta) b + \beta \left[ \mu + \delta + \theta q(\theta) \frac{\mu}{r + s + \tau_y} \right]}{1 + \frac{\beta \theta q(\theta)}{r + s + \tau_y}}.
\end{align*}
\]

(10)

Employed workers receive a pay between income during job search \( b \) if \( \beta = 0 \) and the total revenues generated by the employment \( (\mu) \) if \( \beta = 1 \). Values of \( \beta \) between zero and unity consider a twofold effect of a higher probability of reemployment \( \theta q(\theta) \). First, the lower bound of bargaining outcome increases with \( \theta q(\theta) \) because it is easier to find another vacant job and the threat level is lower that the application for a job is rejected. Second, the upper bound decreases with \( \theta q(\theta) \) because firms have to wait longer until they can fill a vacant job. This reduces the total benefits from market activities which can be shared among firms and workers.

Productivity and wages of the workers differ with age. While a vacant job generates zero revenues \( (V = 0) \), otherwise firms would create an infinite number of jobs, the filled vacancy has a positive value for a firm. In consideration of the age-related wages, it then follows from eq. (4) and eq. (5) that the value of employing an older worker is

\[
J_o = \frac{(1 - \beta)(\mu + (\beta - 1)b)}{r + s + \tau_o + \theta b},
\]

(12)

whereas the younger worker generates a value of

\[
J_y = \frac{(1 - \beta)(\mu + \delta) + (\beta - 1)b}{r + s + \tau_y + \theta b}.
\]

(13)

The dissimilarity in the equations implies that firms may prefer to meet a younger or an older job seeker if they have a vacant job. One age group can have a higher productivity-wage ratio or a lower quitting rate so that it is more valuable to hire workers from this age group. However, firms
also know that search will continue and cause further costs if they refuse a job candidate. The candidates are stochastically drawn from the pool of unemployed and are younger at the probability \( p \) and older at the probability \( (1 - p) \). If the drawing brings the inferior candidate and the firm rejects an employment, the firm expects to pay \( \gamma \) over an additional period. Therefore, firms will accept the first applicant for work as long as extra costs of rejection are equal to the extra gain through employing a superior worker. In this case the expected value of a vacancy is zero because waiting is worthless. In other words, a firm is indifferent between hiring a younger or an older worker if \( V \) according to eq. (6) is equal to zero. This holds true if \( J = \gamma / q(\theta) \) and with eq. (7), the equation for the expected \( J \), we have:

\[
J_o = \frac{1}{1 - p} \left( \frac{\gamma}{q(\theta)\alpha_o} - p \frac{u_y}{w_o} J^y \right).
\] (14)

This is the condition for a pooling equilibrium in which vacancies are open for both age groups. Only the market tightness is variable and guarantees the identity of eq. (14). Rearranging yields the job creation condition:

\[
\frac{1}{q(\theta)} = \frac{(1 - p) \alpha_o}{\gamma} \frac{\mu - w_o}{r + s + \tau_o} + \frac{p \alpha_y}{\gamma} \frac{\mu + \delta - w_y}{r + s + \tau_y}.
\] (15)

The vacancy-matching ratio \( 1/q(\theta) \) is an indicator for job creation. Firms open more vacancies for a given number of job seekers if \( 1/q(\theta) \) increases. It is obvious that easy search and high profits foster job creation.

Steady state equilibrium \((\theta^*, u^*, w_o^*, w_y^*)\) satisfies the flow equilibrium (3), the job creation condition (15), and the two wage equations (10) and (11). Job creation and the wage equations yield the market tightness. Together with the BC equilibrium unemployment is fixed.

### 2.5 The Effects of the Age Structure

A change in the age structure, for example less younger workers due to persistent low fertility rates or baby boomers who grow old, will affect equilibrium unemployment if older and younger workers differ in the considered attributes. Hence, we analyze next the comparative static effects of a change in the share of younger workers \( p \) on equilibrium.

From the job creation condition it follows that the market tightness responds to a change in \( p \) according to:

\[
\frac{\partial \left( \frac{1}{q(\theta)} \right)}{\partial p} = \frac{1}{\gamma} \left( \alpha_y J_y - \alpha_o J_o \right).
\] (16)

The willingness to create a vacancy, \( 1/q(\theta) \), decreases (increases) due to a fall in \( p \) if \( \alpha_y J_y > \alpha_o J_o \) (\( \alpha_y J_y < \alpha_o J_o \)). The age structure has no effect on
job creation under the assumption that $J_y/J_o = \tilde{u}_y/\tilde{u}_o$. This means that different age-related effects, such as the separation risk and the productivity differential, cancel out each other. The condition for this is:

$$
\delta = \left( \mu - b - \frac{\beta q(\theta) \theta \gamma}{1 - \beta} \right) \left( 1 - \frac{u_o r + s + \tau_y}{u_y r + s + \tau_o} \right).
$$

(17)

However, it is more gainful to employ a younger than an older worker if $\delta$ is larger than the right-hand side of the equation. Meeting a younger worker is less likely for a firm with a vacant job as soon as soon as their share $p$ decreases. Consequently, fewer firms seek for new employees if $\delta > (\cdot)$ as the firms now expect lower returns to an advertised vacancy. Hence, $\theta$ declines and $1/q(\theta)$ declines. The opposite holds true if $\delta < (\cdot)$ and $\theta$ increases because firms prefer older workers and their share in the labor force $(1-p)$ grows.

As individuals of different age may have different separation rates, the flow equilibrium changes if $p$ varies. The age proportion of younger workers changes the BC of equation (3) in case of differences in the relative unemployment rates due to age-related separation risks, and we have:

$$
\frac{\partial u}{\partial p} = \tau_y - \tau_o.
$$

(18)

A decline in $p$ reduces the average flows in the labor market if younger workers separate more often from jobs. A higher proportion of older workers then reduces the labor turnover and less job-worker pairs have to be matched. It follows from the standard matching technology that, given a constant job creation, a lower total separation risk correspond to less equilibrium unemployment.

Figure 1 shows equilibrium in the vacancy-unemployment space and illustrates the effects which can arise if the age structure influences flows in the labor market and job creation. The steady state condition for unemployment is the BC which is convex to the origin by the properties of the matching technology. As usual, the BC is downward sloping. Unemployment is low if the vacancy rate is high because job seekers find easily new employments. The JC is the curve that maps the job creation condition. Firms prefer a large pool of unemployed because then they find easily appropriate candidates for their vacancies and save search costs. Hence, firms create more jobs if unemployment is high and the JC slopes upwards.

Taking the old equilibrium as a starting point, four different outcomes may occur if the ratio of younger to older workers decreases.\(^5\) The results are denoted as regime (1) to (4) henceforth. In regime (1) older workers increase the mean separation risk and due to an unfavorable productivity-wage ratio

\[^5\] We analyze the case of a decrease in $p$ because this will take place in the coming years in nearly all developed countries.
they are the less preferred by the firms. A growing share of older workers then implies that the BC shifts outwards and the JC rotates clockwise. The result is that unemployment increases clearly but the effect on the vacancy rate is ambiguous. Regime (2) implies that firms still prefer younger workers who become fewer. However, older workers reduce the labor turnover and the BC consequently shifts inwards. From this it follows that less vacant jobs are available, but it is not clear-cut whether this leads to higher unemployment as also fewer job-worker matches get terminated and less people look for a reemployment. Unemployment decreases if the reduced labor turnover is combined with a favorable productivity-wage ratio of older workers. This takes place in regime (3). Finally, firms can intensify job creation because older employees are a superior workers, but it is not clear whether this reduces unemployment if older workers have a high separation risk. The resulting increase in labor turnover is accompanied by more vacancies but the total employment effect in regime (4) is ambiguous.

The four results are also summarized in Table 1. Note that if more older workers in the labor force imply negative (or positive) employment effects, this applies not only to the older but also to the younger workers. The increase or decline in job creation has the same consequence for all job seekers, which can be either higher or lower chances for filling a vacant job.

For the sake of simplicity we assumed in the model that only a productivity differential and age-related separation risks distinguish younger
Table 1: The effects of a decline in $p$

<table>
<thead>
<tr>
<th>Regime</th>
<th>$\delta &gt; (\cdot)$, $\tau_y &lt; \tau_o$</th>
<th>$\delta &gt; (\cdot)$, $\tau_y &gt; \tau_o$</th>
<th>$\delta &lt; (\cdot)$, $\tau_y &lt; \tau_o$</th>
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<td>$i$</td>
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<tr>
<td>(4)</td>
<td>$o$</td>
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<td>$\sim$</td>
<td>$+$</td>
</tr>
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</table>

$o=$outward, $i=$inward, $l=$left, $r=$right, $+=increase, -=decrease, $\sim=$ambiguous effect

from older workers. However, no general effect is lost by this simplification. This is because other age-related heterogeneity also affects equilibrium either through changes in labor turnover or through changes in the value of a job-worker match. For example one could think of age-related search intensities which affect reemployment probabilities and age-specific unemployment rates. Other examples would be firm specific human capital or seniority which could give older workers higher bargaining power, or differences in the discount rates if younger workers value career progression in a job higher than the current salary. The consequences would be a differential between $J_o$ and $J_y$ which results in more or less job creation if the age structure alters. Hence, more age-related heterogeneity could be captured but the considered effects represent the general impact of the age-structure and the distinction between the four regimes remains untouched by any extension of the model.

3 Empirical Analysis

The age composition of the unemployed changes similarly compared to the age composition of the working age population. There are several possibilities for constructing a proxy variable for the change in the age structure. In our definition young persons are between 15 and 39 years, and old people are between 40 and 64 years old. Exceptions are Australia, Finland, Norway and Sweden. In these countries the cut is at 35 years due to data availability. The following two figures show the different experiences with a changing age composition in the countries we are going to analyze in this section.

In all considered countries the share of the young unemployed decreases at the latest since the middle of the 80s. The exception to this observation is Japan, where population aging was already observable in the late 60s. The enormous share of young unemployed in Australia, Norway, and Spain is remarkable just as the comparatively low share of young unemployed in Germany since the 90s.
3.1 Identification

In this section we present our identification strategy to link the theoretical model with the econometric model. The core equations in section 2 are the BC and the JC. In the first case the unemployment rate \( u \) is a function of the vacancy rate \( v \), the idiosyncratic \( s \) and conditional \( (\tau_y, \tau_o) \) separation rates, the age structure \( p \), and the matching efficiency \( m \). In the second case the market tightness \( \theta \) is a function of income during unemployment \( b \), bargaining power \( \beta \), productivity \( \mu, \delta \), interest rate \( r \), search costs \( (\gamma) \), and the parameters that define the matching rate according to the BC, too:

\[
 u = f(v, s, \tau_y, \tau_o, p, m) \\
 \theta = g(b, \beta, \mu, \delta, r, \gamma, s, \tau_y, \tau_o, p, m)
\]
Our proxy for \( p \) is the share of unemployed who are between 15 and 39 years old. In principle, this variable should capture the differences of a value of a match with a young or an older worker that arise from a change in the age composition. In the first instance this refers to \( \tau_y, \tau_o \) and \( \delta \). However, if trade unions have different bargaining strategies with respect to the age groups or the discount rate is different for the two age groups, this will be captured by \( p \), too. Furthermore, income during unemployment can have components in addition to insurance benefits, which may be different for younger and older job seekers. That is, \( p \) controls for unobserved heterogeneity as a result of a changed share of young workers. To make sure that \( p \) does not capture effects from a changed (effective) participation of older workers, for a given age structure, we additionally control for this effect by considering the employment population ratio of the age group 40 to 64 years (\( \text{epro} \)) in the estimates. A larger \( \text{epro} \) indicates a lower separation rate as workers remain employed when they grow older. Hence, we expect that the BC then shifts inwards and the JC rotates counterclockwise.

The idiosyncratic shock rate in the BC will be approximated by the real interest rate (\( r \)) and the GDP growth (\( gdp \)). In contrast to the BC the shock rate in the JC is approximated by \( r \) and the real import prize for oil in national currencies (\( \text{oil} \)). Since the substitutability for energy in the production process is practically zero in the short run, idiosyncratic shocks increase with the oil prize. The GDP comprises demand and cyclical components, in contrast to the oil prize, which is not a crucial factor for job creation.

We proxy \( \mu \) not only by the real labour productivity but also by labor costs in real productivity units (\( cp \)). The reason for doing so is that \( w \) does not appear in the theoretical model because it is fully explained by \( b \) and \( \beta \). From an empirical perspective this approximation is insufficient and may cause a bias especially on the parameters that measure the effect of \( p \). However, we consider the bargaining power of workers approximated by union density (\( ud \)) and bargaining coverage (\( bc \)) to map effects beyond current wage determination. Generally speaking, we expect negative effects on JC. Nevertheless, positive effects are possible if unions negotiate more for the unemployed than for insiders and bargaining coverage stabilize wages at the firm level. However, due to the explicit consideration of the wage-productivity ratio, the parameters for bargaining power reflect rather expectations on future wages and bargaining shocks than bargaining as a whole, and the estimated effects are expected to decrease in magnitude.

The vacancy rate and the benefit replacement rate are directly observable. Furthermore, our proxy for \( r \) is the real short term interest rate. The search costs are expected to be constant to a greater or lesser extent but different across countries. Therefore, the constant and the country fixed effects take them into account.

The matching efficiency, which is taken exogenously in the theoretical
model, is a function of benefit replacement rate and employment protection \((ep)\) in the econometric model. A change in the replacement rate may affect the search behavior of the job searcher. Two effects are possible: First, a higher replacement rate extends the search for a better match because search is less costly. In this case the resulting higher match quality decreases the consequent quit rate which outweighs the increased search duration. As a result the matching efficiency increases. Second, the search duration effect outweighs the quit rate effect and the matching efficiency decreases. Similarly, a change in employment protection may affect the search behavior of the firms. Again, two effects are possible: First, since the costs of dismissals increase, firms intend to reduce dismissals and select candidates more cautiously. In total, the matching efficiency increases if the resulting lower separation rate effect exceeds the effect of extended duration of search. In the second case the opposite occurs and the matching efficiency decreases.

In addition to the variables that explained BC so far, we control for institutional effects on job destruction and therefore on additional shifts of the BC. The labor market institutions that are expected to influence job destruction are union density, bargaining coverage, and employment protection. The three institutions reduce the flexibility of firms to react to changes in the economic environment and are consequently expected to increase job destruction. As a general macro economic effect on job destruction GDP growth is considered as a proxy, too.

It is worth noting that the endogenous matching efficiency in the estimates can produce both directions of effects of \(ep\) and \(b\) on BC and JC. Therefore, \(ep\) may capture opposing effects on the BC as well as \(b\) on the JC, because they are also proxies for job destruction and benefit respectively. Finally, the reduced form of the BC and the JC are:

\[
    u = f(v, b, ep, ud, bc, r, gdp, epro, p)
\]

\[
    \theta = g(b, ep, ud, bc, r, cp, oil, epro, p)
\]

### 3.2 Data

The empirical analysis comprises data on 12 OECD Countries: Australia, Canada, Finland, France, Germany, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, and the USA. The period is 1977 to 1999 for the BC and 1978 to 1999 for the JC. The selected periods yield a balanced panel and include the turning point during the mid 70s and early 80s when the ratio of younger to older job seekers began to decrease substantially in most considered countries.

From OECD online database we take the standardized unemployment rate, the GDP, and data on unemployment, employment, and population for different age groups. The data on labor market institutions and vacan-
cies\textsuperscript{6} are taken from Nickell and Nunciata (2002) and Baker et al. (2002). The institutions include benefit replacement rate, employment protection, bargaining coverage, and net union density. For the construction of these data we refer to Nickell and Nunciata (2002). Data on the real short term interest rate and labor costs are also taken from Nickell and Nunciata (2002) and the OECD online database. The data on labour productivity are taken from the online service of the US department of Labor, bureau of labor statistics. Finally, the time series on exchange rates are taken from the US board of governors of the Federal Reserve System and data on the actual paid oil price for imported oil into the US are taken from the Energy Information Administration of the US government.

3.3 Econometric Model

The econometric specification of the BC and the JC is basically that of the reduced form. Since institutions typically have low variation and are highly correlated within a country, each equation will be estimated as panel model. However, the effects of the age structure are estimated country specific because we can distinguish between four regimes of different effects of $p$ and there is no reason that all countries share the same regime. The lagged dependent variable is also considered on the right hand side to focus on the short run effects of the exogenous variables. Furthermore, we choose the park estimator to control for contemporaneous correlation across the countries and country specific heteroskedasticity. In this case only country fixed effects are allowed. The time fixed effects are approximated by a linear time trend in each equation. The econometric equations are:\textsuperscript{7}

$$
\log (u_{it}) = \beta_0 + \beta_1 \log (u_{it-1}) + \beta_2 \log (v_{it}) + \beta_3 b_{it} \\
+ \beta_4 e_{p_{it}} + \beta_5 u_{it} + \beta_6 b_{c_{it}} + \beta_7 r_{it} + \beta_8 g_{dp_{it}} \\
+ \beta_9 e_{pro_{it}} + \beta_{10} \text{trend}_{it} + \kappa_i p_{it} + \phi_i + \epsilon_{1it}
$$

$$
\log (\theta_{it}) = \alpha_0 + \alpha_1 \log (\theta_{it-1}) + \alpha_2 b_{it} + \alpha_3 e_{p_{it}} \\
+ \alpha_4 u_{it} + \alpha_5 b_{c_{it}} + \alpha_6 r_{it} + \alpha_7 c_{p_{it}} + \alpha_8 o_{il_{it}} \\
+ \alpha_9 e_{pro_{it}} + \alpha_{10} \text{trend}_{it} + \lambda_i p_{it} + \psi_i + \epsilon_{2it}
$$

The estimated effects of the age composition, $\kappa_i$ and $\lambda_i$, reveal how unemployment changes due to shifts of and moves on the BC. Furthermore, they identify the regime according to Table 1.\textsuperscript{6}

\textsuperscript{6}It is worth noting that the national official statistics report only a fraction of vacancies. However, it is not possible to account for this problem for each country. Therefore, estimates have to be interpreted carefully.

\textsuperscript{7}The use of the logarithm of $p$ would estimate the wrong functional form if the parameter is positive but less than one. In this case the relationship between unemployment and aging is a monotonic increasing concave function. This would be contradictory to the theoretical model with a monotonic increasing convex function.
3.4 Results

Table 2 shows the estimation results. The dependent variables are the standardized unemployment rate in eq. (19) and the market tightness in eq. (20). We can conclude on the basis of two unit root tests that the panel is cointegrated. Rightward the parameters we provide in parenthesis the t-statistics. Probability values are provided in parenthesis for the unit root tests.

First we discuss the cross country effects. The lagged dependent variables and the vacancy rate have the expected effects. Concerning the BC, the negative effects of $b$ and $ep$ indicate that the matching efficiency increases with these variables. It is remarkable that this effect applies especially to search at the firm side, since the parameter for $ep$ reflects the net effect of a change in the matching efficiency and job destruction. On the other hand, we find that job destruction increases with trade union power and economic slowdown, whereas the level of wage bargaining has no significant effect. Regarding the proxies for a productivity shock it is again the GDP growth that is significant.\(^8\) The real interest rate does not significantly affect unemployment at given GDP. Finally, unemployment reduces if $epro$ is higher. This is as expected because we see $epro$ as an indicator for the job duration of older workers.

Employment protection leads to a positive effect on JC through an increase in the matching efficiency, whereas the effect of unemployment benefits is not significant as the expected negative direct effect of $b$ is small. The labor costs in productivity units have no significant effect across all considered countries. That is, on average wages increased moderately compared to productivity. However, the variables that capture bargaining indicate that a more flexible level of negotiations would increase JC. As expected, a rise in the interest rate and in the oil price lowers JC. The extended duration of matches yields that $epro$ has positive effects. Not surprisingly, the interest rate is significant in the JC, since it has a direct and an indirect effect that are expected to have the same direction.\(^9\)

The trend variables can be interpreted as an autonomous increase in job destruction in the BC and a rise in search costs for firms in the JC. That is, search costs differ across countries and change over time. Likewise it is possible that both trends reflect an autonomous decrease in the matching efficiency.

The age structure has effects on flows in the labor market and the expected value of a match between jobs and workers if the effects measured by

\(^8\)Variables that act twice but unidirectional as proxy are expected to be significant for both channels by definition.

\(^9\)The estimated institutional effects differ from those of Nickell et al. (2005), however, generally speaking they estimate a reduced form of our specification. In addition, their period and set of considered countries differ from our sample, too.
### Table 2: Estimation results for BC and JC

<table>
<thead>
<tr>
<th></th>
<th>BC</th>
<th>JC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant</strong></td>
<td>1.578 (5.486)</td>
<td>-3.945 (-3.657)</td>
</tr>
<tr>
<td><strong>log(u_{it-1})</strong></td>
<td>0.583 (22.173)</td>
<td></td>
</tr>
<tr>
<td><strong>log(q_{it-1})</strong></td>
<td>0.593 (13.541)</td>
<td></td>
</tr>
<tr>
<td><strong>log(v_{it})</strong></td>
<td>-0.216 (-22.169)</td>
<td></td>
</tr>
<tr>
<td><strong>b_{it}</strong></td>
<td>-0.087 (-1.117)</td>
<td>0.272 (0.941)</td>
</tr>
<tr>
<td><strong>ud_{it}</strong></td>
<td>1.243 (7.334)</td>
<td>-0.025 (-0.049)</td>
</tr>
<tr>
<td><strong>bc_{it}</strong></td>
<td>0.025 (1.443)</td>
<td>-0.106 (-2.010)</td>
</tr>
<tr>
<td><strong>ep_{it}</strong></td>
<td>-0.29 (-3.620)</td>
<td>1.966 (6.771)</td>
</tr>
<tr>
<td><strong>r_{it}</strong></td>
<td>-0.095 (-0.660)</td>
<td>-1.551 (-2.350)</td>
</tr>
<tr>
<td><strong>gdp_{it}</strong></td>
<td>-1.615 (-13.301)</td>
<td></td>
</tr>
<tr>
<td><strong>oil_{it}</strong></td>
<td></td>
<td>-0.112 (-4.933)</td>
</tr>
<tr>
<td><strong>cp_{it}</strong></td>
<td></td>
<td>-0.017 (-0.077)</td>
</tr>
<tr>
<td><strong>epro_{it}</strong></td>
<td>-2.25 (-8.261)</td>
<td>4.499 (4.936)</td>
</tr>
<tr>
<td><strong>linear trend</strong></td>
<td>0.014 (9.106)</td>
<td>-0.019 (-3.236)</td>
</tr>
</tbody>
</table>

- **Australia**: -0.562 (-2.703) 0.663 (0.607)
- **Canada**: 2.057 (7.635) 2.468 (2.220)
- **Finland**: 0.696 (2.040) -1.83 (-2.019)
- **France**: -1.373 (-4.452) 3.024 (2.416)
- **Germany**: -0.703 (-2.689) -4.797 (-6.961)
- **Japan**: -2.247 (-4.815) 1.197 (0.674)
- **Netherlands**: -1.144 (-2.593) -7.015 (-4.193)
- **Norway**: -2.678 (-4.000) -1.434 (-0.665)
- **Portugal**: -1.186 (-3.895) 0.364 (0.355)
- **Spain**: 3.581 (3.908) -17.811 (-4.234)
- **Sweden**: 0.905 (3.223) -4.672 (-6.673)
- **USA**: 1.972 (4.836) 2.021 (1.367)

**if the share of the young unemployed (p) increases**

<table>
<thead>
<tr>
<th></th>
<th>BC</th>
<th>JC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed effects</strong></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td><strong>Adj. R^2</strong></td>
<td>0.993</td>
<td>0.977</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>276</td>
<td>264</td>
</tr>
<tr>
<td><strong>Unit root tests</strong></td>
<td>Levin, Lin &amp; Chu: -9.725 (0.000)</td>
<td>-12.105 (0.000)</td>
</tr>
<tr>
<td></td>
<td>ADF - Fisher: 125.246 (0.000)</td>
<td>162.407 (0.000)</td>
</tr>
</tbody>
</table>
Changes in the age structure with an increasing share of older workers results in an increase (reduction) in unemployment if \( \kappa_i < 0 \) and \( \lambda_i > 0 \) (if \( \kappa_i > 0 \) and \( \lambda_i < 0 \)), with ambiguous effects on the vacancy rate. This corresponds to the first (third) regime\(^{10}\) which implies an outward (inward) shift of the BC and a clockwise (counterclockwise) rotation of the JC. In the other two regimes, changes in BC and JC have opposite effects on unemployment. The total effect depends not only on the magnitude of the two effects, but also on the curvature of the BC and the locus of the initial equilibrium point on the BC.

The results with respect to \( p \) are the following: As a matter of principle job destruction changes significantly in all considered countries if the age structure changes. "Aging friendly" shifts of the BC are estimated for Canada, Finland, Spain, Sweden, and the USA. Regarding the other countries, we expect that flows in the labor market produce higher unemployment in the future due to the ongoing age composition effects. Concerning job creation we find that Finland, Germany, the Netherlands, Norway, Spain, and Sweden may experience positive effects because of future changes in the age composition that we expect from the demographic change, whereas the others will have rather less job creation. If we merge the results, we identify regime (1) for Australia, France, Japan, and Portugal. Workers will experience increasing unemployment spells in these countries. Regime (3) is identified for Finland, Spain, and Sweden. Unemployed workers will have a better chance to get reemployed in these countries. For the remaining countries in the regimes (2) and (4), we first have to calculate the net effects before we can conclude whether workers win or lose in terms of reemployment risks as soon as there is a higher proportion of older workers.

### 3.5 Calculation of Net Effects

In this section we summarize the estimates and show our calculation of net effects if effects on unemployment were ambiguous so far. With respect to the two estimated equations we distinguish between a direct and an indirect effect. The direct effect \( \kappa_i \) shifts the BC and the indirect effect \( \lambda_i \) leads to moves along the BC. In some of the cases, in regime two and four precisely, we found opposing effects that result in ambiguous changes in unemployment as a consequence of the observed changes in the age structure. Moreover, it is interesting to know which of the effects dominates the other even if both have the same direction. Table 3 shows the different effects of an increasing share of the older unemployed on the overall unemployment rate. A negative (positive) sign denotes that unemployment reduces (increases). The total effect depends on the direction of the direct and the indirect effect and, if opposing, on their relative magnitude, the curvature of the BC, and the

\(^{10}\) See Figure 1 and Table 1.
locus of the initial equilibrium point on the BC.

The direct effect dominates in all countries of regime (1) and (2). This also applies to Finland, but for the other countries in the regime (3) group, i.e. Spain and Sweden, and in the regime (4) countries it is the other way around. This result is interesting because it means that the "aging friendly" effect dominates in the regimes with opposed effects of job creation and job destruction. A minor exception is Norway, which has opposing effects of equal magnitude.

Table 3: The effects on unemployment if p decreases

<table>
<thead>
<tr>
<th>Country</th>
<th>direct (BC)</th>
<th>indirect (JC)</th>
<th>total</th>
<th>regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>+</td>
<td>&gt;</td>
<td>+</td>
<td>1</td>
</tr>
<tr>
<td>Canada</td>
<td>-</td>
<td>&gt;</td>
<td>+</td>
<td>- 2</td>
</tr>
<tr>
<td>Finland</td>
<td>-</td>
<td>&gt;</td>
<td>-</td>
<td>- 3</td>
</tr>
<tr>
<td>France</td>
<td>+</td>
<td>&lt;</td>
<td>-</td>
<td>- 1</td>
</tr>
<tr>
<td>Germany</td>
<td>+</td>
<td>&lt;</td>
<td>-</td>
<td>- 4</td>
</tr>
<tr>
<td>Japan</td>
<td>+</td>
<td>&gt;</td>
<td>+</td>
<td>1</td>
</tr>
<tr>
<td>Netherlands</td>
<td>+</td>
<td>&lt;</td>
<td>-</td>
<td>- 4</td>
</tr>
<tr>
<td>Norway</td>
<td>+</td>
<td>=</td>
<td>-</td>
<td>0 4</td>
</tr>
<tr>
<td>Portugal</td>
<td>+</td>
<td>&gt;</td>
<td>+</td>
<td>1</td>
</tr>
<tr>
<td>Spain</td>
<td>-</td>
<td>&lt;</td>
<td>-</td>
<td>- 3</td>
</tr>
<tr>
<td>Sweden</td>
<td>-</td>
<td>&lt;</td>
<td>-</td>
<td>- 3</td>
</tr>
<tr>
<td>USA</td>
<td>-</td>
<td>&gt;</td>
<td>+</td>
<td>2</td>
</tr>
</tbody>
</table>

Taking all results into consideration we find that only the regime (1) countries experienced an increase in equilibrium unemployment when the age structure of jobless people changed towards more older people. For all other countries unemployment declined when the share of the older unemployed increased, with the exception of Norway. With respect to our findings for the USA, the results coincide with those of Bleakley and Fuhrer (1997), Katz and Krueger (1999), and Sneddon and Triest (2002). But they contradict those of Shimer (1991) and Bloom and Canning (2004) who find for the USA that a fall in the share of the young increased total unemployment.

4 Conclusions

In this paper, we examined the relationship between the change in the age structure, according to the demographic change, and unemployment by means of both a theoretical and an empirical model. The modeling relates to the literature on search and matching in the labor market with equilibrium unemployment. We extended the standard framework by age-specific
effects which lead to age-related job creation and job destruction. From a theoretical perspective, the effect of an increasing share of older unemployed on unemployment is ambiguous and divides into four possible regimes. In the case that older workers bring more profits to the firms, either because of a higher productivity or a lower separation risk, the firms will respond to an increase in the relative share of this age groups with a permanent increase in the number of vacancies. Unemployment will strictly decrease if this effect on job creation goes in the same direction as the effect of aging on job destruction. However, unemployment goes up with a higher proportion of older job seekers if firms prefer younger workers who also reduce average job destruction. In contrast to this, the total outcome is ambiguous if the effects of job creation and job destruction are opposing. The net effect on employment then depends on the magnitude of the changes.

In the empirical part of the analysis we estimated two equations: The Beveridge curve and the job creation curve. Based on our proxy for aging, which is the ratio of workers younger to older than prime age, we are able to identify the effects of the age structure on unemployment in 12 OECD countries. This approach allowed to calculate the net effect in the theoretically ambiguous cases. We find all four regimes in the data, which were derived in the theoretical model. Taking all estimation results into account, we suppose that an increase in the share of the older unemployed reduced unemployment in Canada, Finland, Germany, the Netherlands, Norway, Spain, Sweden, and the USA. Therefore, as demographic change is ongoing, and if age-related effects remain the same, these countries can expect lower unemployment rates in the future. On the other hand, the results for Australia, France, Japan, and Portugal show that unemployment tends to further increase when the share of the younger job seekers continuously decreases.

Appendix

Wage Determination: Individual with attribute $i$ can be younger $i = y$ or older $i = o$. Firms have information about the worker’s age but wages follow from identical bargaining rules. Workers receive $W_i - U_i$ from a new employment, whereas the firm gets $J_i$. According to Nash bargaining the wage satisfies:

$$w_i = \arg \max (W_i - U_i) \beta J_i^{1-\beta}.$$  \hfill (21)

The first order condition is

$$0 = \beta (W_i - U_i)^{\beta-1} J_i^{1-\beta} \frac{\partial W_i}{\partial w_i} + (1 - \beta) J^{-\beta} (W_i - U_i) \frac{\partial J_i}{\partial w_i}.$$  \hfill (22)

Solving eq. (9) for $W_i$ and differentiation with respect to wages gives:

$$\frac{\partial W_i}{\partial w_i} = - \frac{\partial J_i}{\partial w_i} = \frac{1}{r + s + \tau_i}.$$  \hfill (23)
We use the equation in eq. (22) and have:

\[
\beta \frac{J_i}{r + s + \tau_i} = (1 - \beta) \frac{W_i - U_i}{r + s + \tau_i}.
\]  
(24)

From this we see that the extra value received by a worker is a factor \(\beta/(1-\beta)\) of the value which remains in the firm:

\[
W_i - U_i = \frac{\beta}{1-\beta} J_i.
\]  
(25)

Using the equation in (9), and substituting \(J_i\) for (12) and (13), yields:

\[
w_o = (1 - \beta) r U_o + \beta \mu, \]  
(26)

\[
w_y = (1 - \beta) r U_y + \beta (\mu + \delta). \]  
(27)

Employed workers receive an income that lays between their reservation wage indicated by \(r U_i\) and the full surplus that an employment generates. Both boundary values can be different for older and younger workers.

From eq. (4) and eq. (5) we can see that firms evaluate an employment according to \(J_o = (\mu - w_o) / (r + s + \tau_o)\) and \(J_y = (\mu + \delta - w_y) / (r + s + \tau_y)\). This and (25) yields:

\[
W_o - U_o = \frac{\beta \mu - w_o}{1 - \beta r + s + \tau_o},
\]  
(28)

\[
W_y - U_y = \frac{\beta \mu + \delta - w_y}{1 - \beta r + s + \tau_y},
\]  
(29)

Finally, with (8) we have:

\[
r U_o = b + \theta q(\theta) \frac{\beta \mu - w_o}{1 - \beta r + s + \tau_o}, \]  
(30)

\[
r U_y = b + \theta q(\theta) \frac{\beta \mu + \delta - w_y}{1 - \beta r + s + \tau_y}. \]  
(31)

From plugging eq. (30) into eq. (26) and eq. (31) into eq. (27) we get the wage equation presented in the text:

\[
w_o = \frac{(1 - \beta) b + \beta \left[ \mu + \theta q(\theta) \frac{\mu}{r + s + \tau_o} \right]}{1 + \beta \theta q(\theta) \frac{1}{r + s + \tau_o}}, \]

\[
w_y = \frac{(1 - \beta) b + \beta \left[ \mu + \delta + \theta q(\theta) \frac{\mu}{r + s + \tau_y} \right]}{1 + \beta \theta q(\theta) \frac{1}{r + s + \tau_y}}.
\]
5 References


OECD Online Database, http://www.oecd.org/statsportal/0,2639,en_2825_293564_1_1_1_1_1_1,00.html


